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Solving Transportation Problems with Hexagonal Fuzzy Numbers Using Best Candidates Method and Different Ranking Techniques

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ABSTRACT
In this paper, we introduce a Fuzzy Transportation Problem (FTP) in which the values of transportation costs are represented as hexagonal fuzzy numbers. We use the Best candidate method to solve the FTP. The Centroid ranking technique is used to obtain the optimal solution.
Keywords: Fuzzy transporation problem, Hexagonal fuzzy number, Best candidate method and centroid ranking.

I. PRELIMINARIES:

1.1. Definition (Fuzzy set [FS]):[8] [3]

Let X be a nonempty set . A fuzzy set Ã of X is defined as Ã = $\{(x,\mu_{\tilde{A}}(x))/x \in X\}$ where $\mu_{\tilde{A}}(x)$ is called the membership function which maps each element of X to a value between 0 and 1.

1.2. Definition (Fuzzy Number [FN]):[8]

A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible value has its weight between 0 and 1. This weight is called the membership function.
A fuzzy number is a convex normalized fuzzy set on the real line R such that:
- There exist at least one $x \in R$with $\mu_{\tilde{A}}(x) = 1$
- $\mu_{\tilde{A}}(x)$ is piecewise continuous

1.3. Definition (Triangular fuzzy numbers [TFN]): [8]

For a triangular fuzzy number A(x), it can be represented by A (a, b, c; 1) with the membership function μ(x) is given by

$$\mu(x) = \begin{cases} \frac{(x-a)}{(b-a)} & , a \le x \le b \\ 1 & , x = b \\ \frac{(c-x)}{(c-b)} & , c \le x \le d \\ 0 & , \text{otherwise} \end{cases}$$

1.4. Definition (Trapezoidal fuzzy numbers [TrFN]): [8]

A Fuzzy number Ã defined on the universal set of real numbers R denoted as Ã = (a, b, c, d; 1) is said to be a trapezoidal fuzzy number if its membership function $\mu_{\tilde{A}}$= (x) is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)} & , a \le x \le b \\ 1 & , b \le x \le c \\ \frac{(d-x)}{(d-c)} & , c \le x \le d \\ 0 & , \text{otherwise} \end{cases}$$

1.5. Definition (Hexagonal fuzzy numbers [HFN]): [5] [8]

A Fuzzy number A_H is denoted by $A_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ where a_1, a_2, a_3, a_4, a_5 and a_6 the real numbers and its membership function are given below:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{1}{2}\left(\frac{(x-a_1)}{(a_2-a_1)}\right) & , a \le x \le b \\ \frac{1}{2}+\frac{1}{2}\left(\frac{(x-a_2)}{(a_3-a_2)}\right) & , a \le x \le b \\ 1 & , a \le x \le a \\ 1-\frac{1}{2}\left(\frac{(x-a_4)}{(a_5-a_4)}\right) & , a \le x \le b \\ \frac{1}{2}\left(\frac{a_6-x)}{(a_6-a_5)}\right) & , a \le x \le b \\ 0 & , \text{Otherwise} \end{cases}$$

2. RANKING OF HEXAGONAL FUZZY NUMBER: [7] [9]

The parametric methods of comparing fuzzy numbers, especially in fuzzy decision making theory are more efficient than non-parametric methods. Cheng's centroid point method **[6],** Chu and Tsao'smethod **[7]**, Abbasbandy and Assady's**[1]** sign-distance method was all non-parametric and was applicable only for normal fuzzy numbers. The non-parametric methods for comparing fuzzy numbers have some drawbacks in practice.

$\tilde{A}_c$is a scaling of fuzzy number defined by

$$\tilde{A}_c = \frac{3\sqrt{3}}{4} \left[(a_1 + a_3 + a_6)\, k + (a_2 + a_4 + a_5) \quad 1 - k\right]$$

3. THE BEST CANDIDATES METHOD (BCM) HAS THE FOLLOWING SOLUTION STEPS: [1][2]

Step1:

Prepare the BCM matrix. If the matrix unbalanced, we balance it and don't use the added row or column candidates in our solution procedure.

Step2:

Select the best candidates that are for minimizing problems to the minimum cost, and maximizing profit to the maximum cost. Therefore, this step can be done by electing the best two candidates in each row. If the candidate repeated more than two times, then the candidate should be elected again. As well as, the columns must be checked such that if it is not have candidates so that the candidates will be elected for them. However, if the candidate is repeated more than one time, the elect it again.

Step3:

Find the combinations by determining one candidate for each row and column, this should be done by starting from the row that have the least candidates, and then delete that row and column. If there are situations that have no candidate for some rows or columns, then directly elect the best available candidate. Repeat Step 3 by determining the next candidate in the row that started from. Compute and compare the summation of candidates for each combination. This is to determine the best combination that gives the optimal solution.

4. PROPOSED METHOD: [1][2]

In this study, we proposed a new solving method for transportation problems by using BCM. The proposed method must operate the as following:

Step1:

We must check the matrix balance, if the total supply is equal to the total demand, then the matrix is balanced and also apply Step 2. If the total supply is not equal to the total demand, then we add a dummy row or column as needed to make supply is equal to the demand. So the transportation costs in this row or column will be assigned to zero.

Step2:

Appling BCM to determine the best combination that is to produce the lowest total weight of the costs, where is one candidate for each row and column.

Step3:

Identify the row with the smallest cost candidate from the chosen combination. Then allocate the demand and the supply as much as possible to the variable with the least unit cost in the selected row or column. Also, we should adjust the supply and demand by crossing out the row/column to be then assigned to zero. If the row or column is not assigned to zero, then we check the selected row if it has an element with lowest cost comparing to the determined element in the chosen combination, then we elect it.

5. Numerical example

Consider the following problem with hexagonal fuzzy numbers:

	D1	D2	D3	Supply
S1	(3,7,11,15,19, 24)	(3,5,7,9,10,12)	(11,14,17,21,25,30)	(7,9,11,13,16,20)
S2	(3,5,7,9,10,12)	(5,7,10,13,17,21)	(7,9,11,14,18,22)	(6,8,11,14,19,25)
S3	(7,9,11,14,18,22)	(2,3,4,6,7,9)	(5,7,8,11,14,17)	(9,11,13,15,18,20)
Demand	(6,9,12,15,20,25)	(6,7,9,11,13,16)	(10,12,14,16,20,24)	

Using the ranking given above the given transportation problem can be reduced as follows:

	D1	D2	D3	Supply
S1	54.16	31.44	78.72	50.40
S2	31.44	49.23	53.91	55.20
S3	53.91	20.91	41.31	57.42
Demand	58.33	41.30	63.39	

From the above selected candidates we elect the best among them.

	D1	D2	D3	Supply
S1	54.16	**41.30** 31.44	**9.1** 78.72	~~50.40~~ ~~41.30~~ 0
S2	**55.20** 31.44	49.23	53.91	~~55.20~~ 0
S3	**3.13** 53.91	20.91	**54.29** 41.31	~~57.42~~ ~~54.29~~ 0
Demand	~~58.33~~ ~~3.13~~ 0	~~41.30~~ 0	~~63.39~~ ~~9.1~~ 0	

The optimal solution using best candidate method is given as follows:

(31.44)*(41.30) + (78.72)*(9.1) + (31.44)*(55.20) + (53.91)*(3.13) + (41.31)*(54.29) =6161.77

6. RANKING OF HEXAGONAL FUZZY NUMBERS: [6] [5] [8]

6.1. Proposed centroid ranking Method:

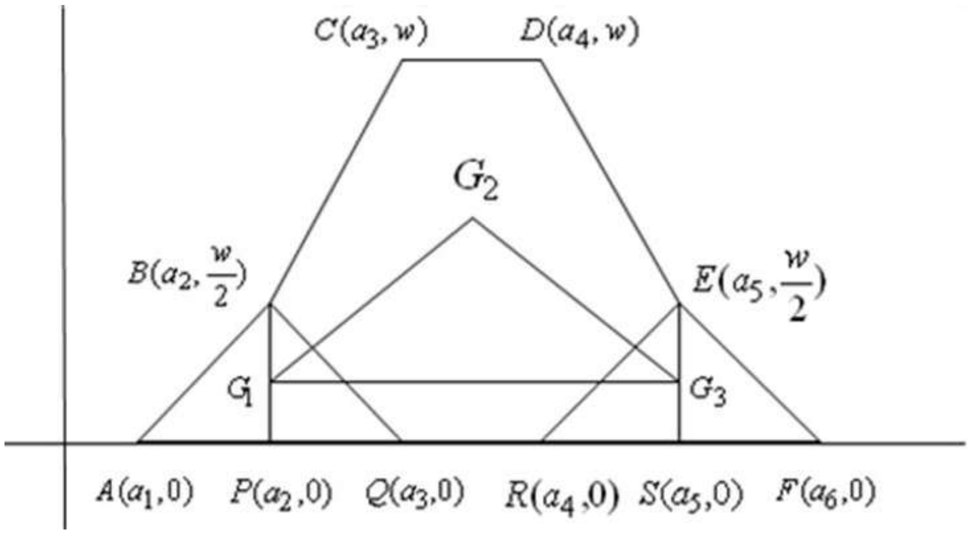

Fig.1 Generalized hexagonal fuzzy number

The centroid of a hexagonal fuzzy number is considered to be the balancing point of the hexagon (Fig.1).Divide the hexagonal into three plane figures .These three plane figures are a Triangle ABQ, Hexagon CDERQB and again a triangle REF respectively. The circumcenter of the centroids of these three plane figures is taken as the point of reference to define the ranking of generalized Hexagonal fuzzy numbers. Let the centroid of the three plane figures be G1, G2, G3, respectively.

The centroid of the three plane figures is $G_1 = \left(\frac{a_1+a_2+a_3}{3}, \frac{w}{6}\right)$, $G_2 = \left(\frac{a_2+2a_3+2a_4+a_5}{6}, \frac{w}{2}\right)$ and $G_3 = \left(\frac{a_4+a_5+a_6}{3}, \frac{w}{6}\right)$ respectively.

Equation of the line G_1, G_3 is $y = \frac{w}{6}$ and G_2 does not lie on the line G_1, G_3. Thus G_1, G_2 and G_3 are non collinear and they form a triangle. We define the centroid $G_{\tilde{A}_H}(x_0, y_0)$ of the triangle with vertices G_1, G_2 and G_3 of the generalized trapezoidal fuzzy number
$\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ as

$$G_{\tilde{A}_H}(x_0, y_0) = \left(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18}, \frac{5w}{18}\right) \text{---------- (1)}$$

The ranking function of the generalized hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ which maps the set of all fuzzy numbers to a set of real numbers is definedas:

$$R(\tilde{A}_H) = (x_0)(y_0) = \left(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18}\right)\left(\frac{5w}{18}\right) \text{-------- (2)}$$

This is the area between the centroid of the centroids $G_{\tilde{A}_H}(x_0, y_0)$ as defined in (1) and (2) the original point.

The mode of the generalized hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ is defined as

$$\text{Mode} = \frac{1}{2}\int_0^w (a_3 + a_4)dx \text{--------------- (3)}$$

The divergence of the generalized hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ is defined as

$$\text{Divergence} = \int_0^w (a_6 - a_1)dx \text{-------------- (4)}$$

The left spread of the generalized hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ is defined as
Left spread $ls = \int_0^{\frac{w}{2}} (a_2 - a_1)dx + \int_{\frac{w}{2}}^{w} (a_3 - a_2)dx$

$$ls = \int_0^w (a_3 - a_1)dx \text{-----------------} (5)$$

The right spread of the generalized hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ is defined as

Right spread $rs = \int_0^{\frac{w}{2}} (a_6 - a_5)dx + \int_{\frac{w}{2}}^{w} (a_5 - a_4)dx$

$$rs = \int_0^w (a_6 - a_4)dx \text{-----------------} (6)$$

6.2. Numerical example:

Consider the following problem with hexagonal fuzzy numbers:

	D1	D2	D3	Supply
S1	(3,7,11,15,19, 24)	(3,5,7,9,10,12)	(11,14,17,21,25,30)	(7,9,11,13,16,20)
S2	(3,5,7,9,10,12)	(5,7,10,13,17,21)	(7,9,11,14,18,22)	(6,8,11,14,19,25)
S3	(7,9,11,14,18,22)	(2,3,4,6,7,9)	(5,7,8,11,14,17)	(9,11,13,15,18,20)
Demand	(6,9,12,15,20,25)	(6,7,9,11,13,16)	(10,12,14,16,20,24)	

Using the ranking given above the given transportation problem can be reduced as follows:

	D1	D2	D3	Supply
S1	3.64	2.15	5.42	3.47
S2	2.15	3.36	2.95	3.75
S3	2.95	1.42	2.82	3.96
Demand	3.96	2.84	4.38	

From the above selected candidates we elect the best among them.

	D1	D2	D3	Supply
S1	**0.21** 3.64	**2.84** 2.15	**0.42** 5.42	~~3.47~~ ~~3.26~~ ~~0.42~~ 0
S2	**3.75** 2.15	3.36	2.95	~~3.75~~ 0
S3	2.95	1.42	**3.96** 2.82	~~3.96~~ 0
Demand	~~3.96~~ ~~0.21~~ 0	~~2.84~~ 0	~~4.38~~ ~~3.98~~ 0	

The optimal solution using best candidate method is given as follows:

= (3.64)(0.26)+(2.15)(2.84)+(5.42)(0.42)+(2.15)(3.75)+(2.82)(3.96) = 28.3765

7. Conclusion:

In this paper, the transportation costs, sources and demands are considered as fuzzy quantifiers which are represented by Hexagonal fuzzy numbers. By using Best Candidate method, the numerical shows that we can get the optimal total cost. The fuzzy transportation problem canbe transformed into crisp transportation problem by using the centroid ranking of fuzzy numbers. This gives a way for solving various fuzzy transportation problems in a simpler form.

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